Comparative Analysis of Multiuser Detection Techniques in SDMA-OFDM System over the Correlated MIMO Channel Model for IEEE 802.16n

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Abstract- Space-Division Multiple Access-OFDM based wireless communication has the potential to significantly increase the spectral efficiency, system performance and number of users. Research in the development of efficient Multiuser Detection (MUD) algorithms for such systems has generated much interest in recent years. The linear Minimum Mean Square Error (MMSE) MUD processes with low complexity but it is limited by its poor performance. The maximum likelihood (ML) detection provides the optimal performance, but its complexity increases exponentially with the number of users. The Successive Interference Cancellation (SIC) and QR decomposition (QRD) MUD can be a substitute to ML detection due its low complexity and near optimal performance. However, all these techniques are incapable to detect users in rank deficient scenario, where number of users is more than that of number of receiving antennas. Hence, the Minimum Bit Error Rate (MBER) MUD by minimizing probability of error directly has become a possible alternative to all aforementioned MUDs in rank deficient scenario and also its BER performance is close to ML detector. This research proposes a comparative analysis of all these MUD schemes. In this paper the performance of various MUD techniques is verified for the correlated MIMO channel models based on IEEE 802.16n standard.

Keywords- Multiple Input Multiple Output; Multiuser Detection; Orthogonal Frequency Division Multiplexing; Space Division Multiple Access; Bit Error Rate

I. INTRODUCTION

SDMA is a structural design of multiple input - multiple output (MIMO) wireless communication that allows many subscribers to share a frequency band at the same time, which may avoid the shortage of spectral resources. In the SDMA uplink scheme, each user is equipped with a single antenna and the base station receiver possesses an array of antennas. The multiple users in the SDMA system are differentiated by the unique user's spatial signature at the receiver antenna [1]. SDMA architecture can be configured and deployed for most of the well-known mobile communication architectures such as CDMA (Code division Multiple Access), TDMA (Time Division Multiple Access) and FDMA (Frequency Division Multiple Access). On the other hand, OFDM is a broadband multicarrier modulation scheme that offers high spectral efficiency by splitting a signal into several orthogonal narrowband channels at different. These narrowband orthogonal sub channels experience almost flat fading in radio environment. Thus, OFDM gets resistance Inter Symbol Interference (ISI) and Inter Carrier Interference (ICI) [2]. Hence the combination of OFDM and SDMA is an efficient technique in high data rate transmission scenario required in next generation fixed and mobile applications [1, 3].

Research in the development of efficient signal detection algorithms for SDMA-OFDM systems has generated much interest in recent years, and several detection algorithms have been proposed in the literatures [4-11]. Among the various MUDs, the classical linear MMSE MUDs exhibit low complexity at the cost of a limited performance [1]. In linear detectors, the user’s signals are estimated with the aid of a linear combiner. A common property of all linear techniques is that they do not consider the residual interference caused by remaining users. The high-complexity optimum ML MUD provided here has capable of achieving the best performance with an exhaustive search [1]. However, the complexity of nonlinear ML detector generally avoids its use in practical systems especially with many users and large constellations. The non linear SIC MUD detects users successively by mitigating interference for stronger signals rather than detecting all users at a time [12, 13]. The performance of SIC is better than that of MMSE detector but not up to mark while comparing with ML detector. The QR decomposition using tree search is a most promising algorithm, which can be implement with low complexity and also provides near optimal solution [14, 15].

However, minimizing mean square error (MSE) in MMSE MUD may not essentially give assurance that the BER of the system is also minimized. Hence, in MBER MUD the error-probability or BER is minimized straightforwardly rather than the minimizing mean square error (MSE) [16-18]. In this paper, the MBER MUD weight calculation of SDMA OFDM system is described and compared with all aforementioned MUD techniques. It is also discussed that the MBER MUD may significantly perform better than all other MUD techniques in rank deficient scenario. In the weight vector updating process the steepest descent gradient algorithm may converge slowly, and a Gauss-Newton algorithm is computationally expensive. Hence, the conjugate gradient method offers a better alternative to find optimal weight vectors while minimizing error gradient by providing low complexity and high convergence speed. In this paper all the MUD techniques are simulated and compared based on the correlated MIMO channel models for IEEE 802.16n standards [19, 20]. This channel models are proposed for scenarios where the cell radius is less than 10 km and the base station (BS) antennas are 15 to 40 m in height.
The organization of the paper is as follows. Section II describes the system models of SDMA OFDM. Section III provides various MUD techniques. Simulation results and analysis are presented in Section IV. Conclusion is given in Section V.

II. SDMA OFDM SYSTEM MODEL

Fig. 1 demonstrates the uplink transmission of SDMA-OFDM system model [16]. In this figure, each of the L simultaneous users is equipped with a single transmitting antenna and the base station is equipped with a P element antenna array. This scenario can improve capacity of the system. The received signal \( y[n, k] \) at the \( k \)th subcarrier of the \( n \)th OFDM block can be characterized by the super position of \( L \) independently transmitted user signals. Thus, the received signal corrupted with Additive White Gaussian Noise (AWGN) at each frequency bin can be expressed in vector form can be expressed as:

\[
y = Hx + n
\]  

(01)

Here the indices \([n, k]\) are omitted for the sake of notational convenience. In the Eq. (1), \( y = [y_1, y_2, ..., y_P] \) \( x = [x_1, x_2, ..., x_L] \) and \( n = [n_1, n_2, ..., n_P] \) are the received vector, the transmitted vector and the noise vector with zero mean and variance \( \sigma_n^2 \) respectively. \( H \) is the frequency domain channel matrix given as follows:

\[
H = \begin{bmatrix}
H_{1,1} & H_{1,2} & ... & H_{1,L} \\
H_{2,1} & H_{2,2} & ... & H_{2,L} \\
... & ... & ... & ... \\
H_{P,1} & H_{P,2} & ... & H_{P,L}
\end{bmatrix}
\]  

(02)

where \( H_{P,L} \) is the channel gain between the \( p \)th receive antenna and \( L \)th user link. The \( l \)th (\( l = 1, 2, ..., L \)) column of channel matrix \( H \) is often referred to as the spatial signature of the \( l \)th user across the receive antenna array. The frequency domain channel response in a multipath environment can be expressed as:

\[
H_{p,l} = \sum_{m=1}^{M} h_{p,l}(m) \exp \left( \frac{-j2\pi km}{N_c} \right), \quad k = 1, 2, ..., N_c
\]  

(03)

where \( N_c \) represents number of subcarriers, \( M \) is number of propagation paths and \( h_{p,l} \) is time domain channel gain of link between \( p \)th receiving antenna and \( l \)th user. Further, in the SDMA-OFDM system each user’s signal separately undergoes OFDM modulation.

III. MULTIUSER DETECTION TECHNIQUES

Multiuser detection is one of the receiver design technology for detecting desired user signal by eliminating noise and interference from neighborhood user’s signal. Generally, multiuser system’s receiver suffer from the inter user interference, where a strong user signal source may influence the reception of weak user signal. The effect of interference is more pronounced in SDMA like wireless multiuser communication systems. Multiuser detection techniques are used to overcome this problem. In the detection process the estimated signal vector ‘\( \hat{x} \)’ can be expressed as:

\[
\hat{x} = W^H y
\]  

(04)

where ‘\( W \)’ is the \((LxP)\) dimension weight matrix and ‘\( y \)’ is the received signal vector.

A. Minimum Mean Square Error (MMSE) MUD

The most popular linear MMSE MUD scheme assumes a priori knowledge of noise variance and channel covariance. In this MMSE MUD, the weight matrix ‘\( W \)’ can be expressed by minimizing mean square error, i.e.

\[
\text{MSE} = E \left[ (\hat{x}_l - x_l)^2 \right], \quad \text{where} \quad x_l \text{ is} \ l\text{th user signal} \quad \hat{x}_l \text{ is} \ l\text{th user estimated signal.}
\]  

Thus the weight vector \( w_l \) is expressed as [1]:

\[
w_l = (HH^H + 2\sigma_n^2 I_P)^{-1} H_l
\]  

(05)

where \( (.)^H \) indicates Hermitian and \( I_P \) is \( P \)-dimensional identity matrix, \( w_l \) is \( l\)th column of weight matrix \( W \) and \( H_l \) is \( l\)th column of channel matrix \( H \). In general, the received signal contains residual interference which is not Gaussian distributed due to multiuser interference. But these linear detectors assume that the received signal is corrupted by only by AWGN. Hence, a non-linear detector is essential to mitigate this residual interference.

B. Maximum Likelihood (ML) Detector

The highest-complexity, highest performance optimum ML MUD uses an exhaustive search for finding the most
likely transmitted users \[i\]. For a ML-MUD supporting \(L\) simultaneous transmitting users, a total of \(2^L\) metric evaluations have to be invoked, where \(m\) denotes the number of bits per symbol, in order to detect the \(L\) user symbol vector \(\hat{X}\) that consists of the most likely transmitted symbols of the \(L\) users at a specific subcarrier, which is given by

\[
\hat{x} = \arg\min_u \|y - H\hat{x}\|, \quad u = 1, 2, \ldots, 2^L
\]  

(06)

where \(y\) is the \((P \times 1)\)-dimensional received signal vector and \(H\) is the \((L \times P)\)-dimensional channel matrix. The set \(U\) in (6) constitutes \(2^L\) number of trial vectors.

C. Successive Interference Cancellation MUD

This algorithm provides improved performance at the cost of increased computational complexity \[13\]. Rather than jointly decoding all users at a time, this nonlinear detection scheme first choose the user with the strongest signal-to-noise ratio (SNR) detect first and then cancels its effect from the overall received signal vector. It then proceeds to detect next strongest user. OSIC algorithm is as follows

Initialization:

\(i \leftarrow 1, \quad r_i = r\)

Ordering: Determine the optimal detection order by choosing the row with minimum Euclidian norm (strongest SNR), i.e. most reliable signal.

\[G_i = (H^H H + \sigma^2 I_m)^{-1} H^H\]

\(k_i = \arg \min_j \|G_i \|_2^2\)

Nulling: Estimate the strongest transmit signal by nulling out all the weaker transmit signals.

\[w_{k_i} = (G_i)^T k_i\]

Detection: Detect the transmitted signal identified in the previous step and make a decision.

\[y_i = w_{k_i}^T r_i\]

Slicing: Detect the value of the strongest transmit signal by slicing to the nearest signal constellation value.

\[\hat{a}_{k_i} = Q(y_i)\]

Interference Cancellation: Cancel the effect of the detected signal from the received signal vector to reduce the detection complexity for the remaining signals, i.e. Remove interference from \(\hat{a}_{k_i}\).

\[r_{i+1} = r_i - \hat{a}_{k_i} H k_i\]

Recursion,

\[G_{i+1} = (H^H H + \sigma^2 I_m)^{-1} H^H\]

\[k_{i+1} = \arg \min_{j \neq \{k_i, \ldots, k_L\}} \|G_{i+1}\|_2^2\]

\[i = i + 1\]

D. QR Decomposition based Detector

The QRD-M algorithm provides near ML detection performance with comparatively low complexity \[14\]. It is basically a breadth first tree traversal algorithm. At each detection layer, QRD-M algorithm keeps \(M\) reliable nodes instead of deciding the symbol. Detection is done after processing all layers. The concept of QRD-M is to apply the tree search to detect the symbols in a sequential manner. Starting from the first layer i.e. \(i = P\), the algorithm calculates the metrics for all possible values of \(\hat{y}_i\) from the constellation using Euclidean distance given as (12). The metrics of these points or nodes are then ordered, and only \(M\) nodes with the smallest metrics are retained and the rest of the list is deleted. The same process is applied to the next layer nodes, and this process continues to the last layer i.e. \(i=1\). To achieve near-ML detection performance for QRD-M algorithm, \(M\) should be large enough for the selected paths to include the correct one.

Since the complexity of ML detection depends on the constellation size and number of users, an exhaustive search required. This is infeasible in practical if either the constellation size or the signal dimension size is very large. To overcome such problem the QRD-M algorithm was proposed \[14\]. In this algorithm QR-decomposition of channel matrix is obtained as \(H = QR\), where \(Q\) is a \((P \times P)\) unitary matrix such that \(Q^H Q = I\) and \(R\) is \((P \times L)\) upper triangular matrix. Using Eq. (1)

\[y = Q Rx + n\]

\[Q^H y = Rx + Q^H n\]

\[\tilde{y} = Rx + \tilde{n}\]

\[
\begin{bmatrix}
\tilde{y}_1 \\
\tilde{y}_2 \\
\vdots \\
\tilde{y}_p
\end{bmatrix}
= 
\begin{bmatrix}
R_{1,1} & R_{1,2} & \cdots & R_{1, L} \\
0 & R_{2,2} & \cdots & R_{2, L} \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & R_{p,L}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_L
\end{bmatrix}
+ 
\begin{bmatrix}
\tilde{n}_1 \\
\tilde{n}_2 \\
\vdots \\
\tilde{n}_p
\end{bmatrix}
\]  

(10)

where \(R_{ij}\) is the \((i, j)\) th component of \(R\). The statistical properties of \(n\) and \(\tilde{n}\) are equal. Therefore, the ML detection problem in Eq. (6) can be reformulated as:

\[
\hat{x} = \arg \min \|\tilde{y} - R \hat{x}\|
\]

(11)

\[
= \arg \min \left\{ \sum_{i=1}^{L} \|\tilde{y}_i - \sum_{j=1}^{L} R_{ij} x_j\|^2 \right\}
\]

(12)

Here, \|\| denotes the absolute value. Let us assume

\[
d(\hat{x}) = \sum_{i=1}^{L} \|\tilde{y}_i - \sum_{j=1}^{L} R_{ij} x_j\|^2
\]

(13)
where $d$ is partial Euclidian distance. To account for the case when the decision is made on symbols from $x_p$ to $x_k$, $1 \leq k \leq L$, Eq. (13) can be modified as

$$d_k(x) = \sum_{i=1}^{L} |y_i - \sum_{j=1}^{k} R_{ij} x_j|^{2}$$

where $x = [x_1, x_2, \ldots, x_k]$ of length $L-k+1$.

A fairly large $M$ number of branches are needed to QRD-M scheme in order to approach the MLD performance. The maximum value of $M$ can be up to castellation size of modulation used. For example, for 16-QAM modulated systems, $M$ can take value up to 16.

E. Minimum Bit Error Rate (MBER) MUD

Initially we assume the channel matrix $H$ is explicitly defined. From [16-18], the MBER solution for BPSK modulated signal is defined as the probability of error $P_e$ encountered at the output of the SDMA MUD characterized by the combiner weight vector $w_p$ of user $p$ may be expressed as:

$$P_e(W_i) = \frac{1}{N_b} \sum_{j=1}^{N_b} \left[ \frac{\sigma_n}{\sigma_n^{(j)}} \right] \left[ \frac{\left| w_i^H y_j \right|^2}{\sum_{j=1}^{N_b} \left| \frac{\left| w_i^H y_j \right|^2}{\sigma_n^{(j)}} \right|} \right]$$

Where $N_b$ is the number of equiprobable combinations of the binary vectors of the $P$ users, i.e. $N_b = 2^P$.

$\sigma_n$ is the variance of the noise,

$b_j^{(i)}, j=1,\ldots,N_t$ is the transmitted bit of user $i$, and $b_j^{(i)}, j=1,\ldots,N_b$ constitutes a possible value of the noiseless ($L \times 1$) dimensional received signal vector $y$. The MBER solution is defined as:

$$W_{i(MBER)} = \arg \min_{W_p} P_e(W_i)$$

where $P_e(W_i)$ is the error probability with weight vector $W_i$. In general an iterative strategy based on the steepest descent gradient method may be used for finding the MBER solution. According to this method, the linear SDMA MUD’s weight vector $W_i$ is iteratively updated, taking the MMSE weights as initial weights, until the specific SDMA MUD weight vector that exhibits the lowest BER is arrived. In each step, the weight vector is updated according to a specific step size $\mu$ in the vectorial direction in which the BER cost function decreases most rapidly, in the direction opposite to the gradient of the BER cost function. The BER is independent of the magnitude of the MUD’s weight vector, and hence the knowledge of the orientation of the detector’s weight vector is sufficient for defining the decision boundary of the linear MBER OFDM/SDMA detector. And the cost function is derived as

$$\nabla_{w_i} P_e(W_i) = \frac{1}{N_b \sqrt{2\pi \sigma_n}} \left( \frac{w_i^H w_i - w_i^H y_j}{w_i^H w_i} \right)^{1/2} \exp \left( -\frac{(\sigma_n^{(j)})^2}{2\sigma_n^2 w_i^H w_i} \right) \left[ \frac{\left| \frac{w_i^H y_j}{\sigma_n^{(j)}} \right|^2}{\sum_{j=1}^{N_b} \left( \frac{\left| w_i^H y_j \right|^2}{\sigma_n^{(j)}} \right) } \right]$$

where $s_i^{(j)} = w_i^H y_j$. In this paper conjugate gradient [16] algorithm is used for weight updation, which is given below.

Initialization:

Choose step size $\mu > 0$ and termination scalar $\beta > 0$. Set initial weight $W(1)$ from MMSE detection and $d(1) = -\nabla P_e(W_p(1))$ and iteration number $i = 1$.

Loop:

If $\|\nabla P_e(W(i))\| = \|\nabla P_e(W(i))\| < \beta$ then stop

$$W(i+1) = W(i) + \mu d(i)$$

$$d(i+1) = \phi d(i) - \nabla P_e(W(i+1))$$

$i = i + 1$, go to Loop.

IV. SIMULATION ANALYSIS

In this section, we have analysed the performance of SDMA-OFDM system using the various multiuser detection techniques mentioned in previous section. In the simulation study a 4x4 SDMA OFDM system is considered. Further simulation parameters chosen are outlined in Table I. Performance carried through BER versus SNR in dB plots.

**TABLE I SIMULATION PARAMETERS**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of carriers</td>
<td>128</td>
</tr>
<tr>
<td>Guard Interval</td>
<td>32</td>
</tr>
<tr>
<td>Number of frames</td>
<td>1000</td>
</tr>
<tr>
<td>Number of users</td>
<td>4</td>
</tr>
<tr>
<td>Number of Rx antennas</td>
<td>4</td>
</tr>
<tr>
<td>Modulation Technique</td>
<td>BPSK</td>
</tr>
<tr>
<td>Minimum Acceptable Error in MBER MUD</td>
<td>0.0001</td>
</tr>
<tr>
<td>Optimization Used</td>
<td>Conjugate Gradient</td>
</tr>
<tr>
<td>Step Size</td>
<td>0.0975</td>
</tr>
<tr>
<td>Channel impulse response</td>
<td>Correlated MIMO Channel given in Table II.</td>
</tr>
</tbody>
</table>

**TABLE II CORRELATED MIMO CHANNEL MODEL FOR IEEE 802.16N**

<table>
<thead>
<tr>
<th>Delay</th>
<th>Tap 1</th>
<th>Tap 2</th>
<th>Tap 3</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power (30 antenna)</td>
<td>0</td>
<td>-15</td>
<td>-20</td>
<td>dB</td>
</tr>
<tr>
<td>K-Factor</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Doppler shift</td>
<td>0.4</td>
<td>0.3</td>
<td>0.5</td>
<td>Hz</td>
</tr>
<tr>
<td>Sampling Frequency</td>
<td>0.25</td>
<td>MHz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Antenna correlation</td>
<td>0.4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 2 Channel behaviour of IEEE 802.16n in terms of (a) power spectral density (b) frequency response

Fig. 3 The BER performance of the four different users in an SDMA system employing four receiver antennas under SWATM channel conditions given in Table 1 (a) User 1. (b) User 2. (c) User 3. (d) User 4
In the simulation study the correlated MIMO channel model according to IEEE 802.16n standards is used as given in Table II. The power spectral density plot and frequency response plots are given in Fig. 2. The spectral density measures the frequency content of a stochastic process and helps identify periodicities. From the frequency response we can observe that the channel behaviour for each data samples sent.

Fig. 3(a)-3(d) illustrates a comparative BER performance of four different users in SDMA-OFDM system provided with four receiving antennas under correlated MIMO Channel model as given in Table II. From these curves it is observed that, the linear MMSE MUD detects each user at different SNR values because at the receiver end the stronger user signals effects the weaker signals and it may not allow proper detection of weaker signals.

The average BER performance of four different users determined for correlated MIMO Channel model in an SDMA-OFDM system eqipped with four receiving antennas is shown in Fig. 4. It is inferred from Fig. 4, that being a linear detector the MMSE MUD cannot mitigate multiuser interference adequately, hence it results poor BER performance while ML MUD gives the optimal performance. The BER performance of the nonlinear SIC MUD is better than MMSE MUD and close to sub optimal QR detector. It is also observed that the BER performance of the suboptimal MBER MUD and QRD MUDs are almost equal, better than MMSE MUD and close to ML MUD. More explicitly in Fig. 4, at $10^{-4}$ BER level the MBER MUD has 2 dB and 1 dB SNR gains with respect to SIC detector and MMSE MUD respectively.

Further, a 4x8 SDMA-OFDM system is studied over correlated MIMO channels given in Table II. In Fig. 5, it is seen that as the number of users increases, the BER performance degrades due to the increased multiuser interference. The MMSE, SIC, QRD MUDs in Fig. 5(a) to (c) can only support a maximum number of users that is equal to the number of receiver antennas, which is four in this case. Once the number of users exceeds the number of receiver antennas, these MUDs become incapable of differentiating the users, and performance detorates. Under such case the MBER MUD as shown in Fig. 5(d) performs significantly better by supporting up to eight users. Finally by comparing Fig. 5(a) to (d), we may conclude that the MBER MUD is capable of supporting more users than the number of receiver antennas.

REFERENCES