Abstract - The dynamics model considering payload eccentricity and friction effects of an omni-directional mobile robot is first derived using Lagrange’s equation. Based on the dynamics model with uncertainty, a stable adaptive fuzzy control law is derived using the backstepping method via Lyapunov stability theory. In order to compensate for the model uncertainty, a nonlinear damping term and a fuzzy function approximator are included in the control law, and the parameters adaptation law with $\sigma$-modification is considered for the uncertainty estimation. The proposed control strategy has arbitrary trajectory following capability of simultaneous translation and rotation control for the wheeled robot. Computer simulations are used to illustrate the effectiveness of the suggested control approach.

Keywords - Omni-Wheeled Robot; Dynamics Model; Backstepping Method; Adaptive Fuzzy Control; Lyapunov Stability

I. INTRODUCTION

Mobile robots constructed with three Swedish wheels have good mobility that means they have simultaneous and independent rotational and translational motion capabilities. For examples, they can move in any direction without changing its orientation, or they can freely rotate without translation by suitable control of the three wheels. This omni-directional capability provides them greater flexibility in the applications on dynamic and congested environments. Ould-Khessal et al. [1] and Liu et al. [2] applied them in a robot soccer team design.

Modeling and controller design play key roles on the development and applications of an omni-directional mobile robot. The most common approach to the controller design of an omni-directional robot considers only its kinematics model [e.g. 3-8]. And usually each motor is controlled by an independent PID controller to track the speed commands computed from inverse kinematics [4-6]. Velasco-Villa et al. [7, 8] considered the discrete-time feedback linearization control with Smith predictor compensator for an omni-directional robot subject to communication network time delay. The above kinematic control approaches neglect the dynamics effect and thus would lower the effective moving speed and tracking performance that can be obtained.

Nagy et al. [9, 10] and Samani et al. [11] proposed the kinematics and dynamics models of omni-directional mobile robots, which considered the motor dynamics but neglected the nonlinear coupling between the translational and rotational velocities. The dynamics model can then be approximated as a linear system. Thus, Nagy et al. [9, 10] considered the optimal path planning and developed the position control strategies with considering orientation control. And based on the simplified linear model, Samani et al. [11] proposed two independent PID controllers for the position and orientation controls. Since traditional linear controllers were adopted in these works, their multi-axis arbitrary trajectory tracking performances would not be satisfactory.

D’Andréa-Novel et al. [12] had shown that by means of dynamic state feedback, it is possible for 3-wheeled mobile robots to track arbitrary fast trajectories not reduced to equilibrium points. Based on the dynamics model, Watanabe et al. [13] proposed a control scheme of the resolved-acceleration type with PI and PD feedback, for mobile service robots. Vázquez and Velasco-Villa [14, 15] considered the computed-torque control of an omnidirectional mobile robot based on its dynamics model. Liu et al. [2] proposed a nonlinear controller consisting of an outer-loop (kinematics) controller and an inner-loop (dynamics) controller for an omni-directional mobile robot, which are both designed using the Trajectory Linearization Control (TLC) method based on a nonlinear dynamics model. Chen et al. [16] presented a backstepping control method using sum of squares technique to design a nonlinear controller for a three-wheeled omni-directional mobile robot. Velasco-Villa et al. [17] presented a dynamic trajectory tracking control based on a passive approach, for an omnidirectional mobile robot. Bugeja and Fabri [18] presented an adaptive neural network dynamic control for a nonholonomic mobile robot with two-active and two-passive wheels. Recently, Bugeja and Fabri [19] presented a dual adaptive neurocontroller based on the unscented transform, for the dynamic control of nonholonomic wheeled mobile robots. The robot nonlinear dynamic functions are approximated by a multilayer perceptron neural network trained via an unscented Kalman predictor.

In this work, we will propose a more complete dynamics modeling of an omni-directional mobile robot with three Swedish wheels considering both the friction and load eccentricity effects, and an adaptive fuzzy control is then constructed using the backstepping method via Lyapunov stability theory. After establishing the kinematics model, the 3-DOF dynamics model in the Cartesian space is directly derived using the Lagrange’s equation, which can be used for arbitrary translational and rotational dynamic control. The derived adaptive controller with fuzzy uncertainty compensator has excellent three-axis arbitrary trajectory.
tracking performance, even the platform is encountered a 
not small eccentricity uncertainty. Finally, simulation results 
are presented to illustrate the suggested control system 
performance. The suggested nonlinear controller is more 
complex than traditional PID control, however it could have 
more satisfactory and faster arbitrary tracking capability.

II. MODELING OF AN OMNI-DIRECTIONAL WHEELED ROBOT

A Swedish wheel consists of a main wheel and passive 
rollers attached to the wheel circumference as shown in Fig. 
1, where \( \gamma \) is the angle between the rotation axis of each 
passive roller and the main wheel axis. Complete kinematics and dynamics models of 
an omni-directional robot with three Swedish wheels will be 
considered in this section.

Consider a Swedish wheel mounted on a mobile robot 
with local coordinate frame \( \{R\} \) (\( G - X_s Y_s Z_s \)), as shown 
in Fig. 1, where point \( A \) is the wheel center and other 
geometric parameters are defined as follows. \( \alpha \) is the 
angle of vector \( \overrightarrow{GA} \) relative to \( X_s \) axis, and \( \beta \) is the 
angle between \( \overrightarrow{GA} \) and main wheel axis. The distance 
from \( G \) to wheel center \( A \) is \( l \), and the main wheel’s radius is 
\( r \). And \( \phi \) and \( \phi_{sw} \) are respectively the rotation speeds of 
the main wheel and the passive roller contacting with the 
flat floor.

Assume that the contact point between the Swedish 
wheel and the floor is an instantaneous rotation center, that 
is, it is in pure rolling contact without slipping, then the 
corresponding velocity of wheel center \( A \) is \( r \phi \) along the 
tangential direction as shown in Fig. 1. Thus, the wheel 
center \( A \)’s velocity component along the contact roller’s axis is 
\( r \phi \cos \gamma \).

Assume that the robot’s instantaneous translation 
velocity in terms of local frame \( \{R\} \) is \( [\dot{x}_g \; \dot{y}_g] \), and the 
rotation velocity about \( Z_s \) axis is \( \dot{\theta} \). Then the wheel 
center \( A \)’s velocity vector can also be computed by summing 
the translational velocity components \( \dot{x}_g, \dot{y}_g \), and the 
relative velocity \( l \dot{\theta} \) due to the rotation shown in Fig. 1. 
The wheel center \( A \)’s velocity component along the contact 
roller’s axis can be expressed as \[1\]:

\[
x_g \cos \left[ \frac{\pi}{2} - \left( \frac{\pi}{2} - (\alpha + \beta) \right) - \left( \frac{\pi}{2} - \gamma \right) \right]
+ \dot{y}_g \cos \left[ \frac{\pi}{2} - (\alpha + \beta) + \left( \frac{\pi}{2} - \gamma \right) \right]
+ l \dot{\theta} \cos \left[ \alpha + \left( \frac{\pi}{2} - (\alpha + \beta) \right) + \left( \frac{\pi}{2} - \gamma \right) \right]
= \dot{x}_g \cos \left[ (\alpha + \beta + \gamma) - \left( \frac{\pi}{2} \right) \right]
+ \dot{y}_g \cos \left[ (\alpha + \beta + \gamma) + l \dot{\theta} \cos \left[ (\beta + \gamma) \right] \right]
= \left[ \dot{x}_g \; \dot{y}_g \; \dot{\theta} \right]^T
\]

Thus, we have the constraint equation for a Swedish 
wheel to have no slipping along the contact roller’s axis as:

\[
[\sin (\alpha + \beta + \gamma) - \cos (\alpha + \beta + \gamma) - l \cos (\beta + \gamma)]
\left[ \dot{x}_g \; \dot{y}_g \; \dot{\theta} \right]^T = r \phi \cos \gamma
\]

Since the rotation matrix representing the orientation of 
the inertia frame \( \{I\} \) with respect to the robot frame \( \{R\} \) can 
be expressed as

\[
^R_e \mathbf{R}(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

where \( \theta \) is the angle between axes \( X_s \) and \( X_i \), and 
the robot’s velocity vector in terms of robot frame \( \{R\} \), 
\( \dot{\xi}_g = [\dot{x}_g \; \dot{y}_g \; \dot{\theta}]^T \) can be computed as:

\[
\dot{\xi}_g = ^R_e \mathbf{R}(\theta) \dot{\xi}_i,
\]

where \( \dot{\xi}_i = [\dot{x}_i \; \dot{y}_i \; \dot{\theta}]^T \) is the velocity vector of the robot 
geometric center \( G \) (refer to Fig. 2) in terms of inertia frame \( \{I\} \). And Eq. (2) can be transformed to:

\[
[\sin (\alpha + \beta + \gamma) - \cos (\alpha + \beta + \gamma) - l \cos (\beta + \gamma)]
\left[ \dot{x}_g \; \dot{y}_g \; \dot{\theta} \right]^T = r \phi \cos \gamma
\]

In the direction orthogonal to the contact roller’s axis, 
the motion is not constrained because of the free rotation of 
the passive contact roller, thus we have the following 
velocity relation:

\[
x_g \sin \left[ \frac{\pi}{2} - \frac{\pi}{2} - (\alpha + \beta) - \frac{\pi}{2} - \gamma \right]
- \dot{y}_g \sin \left[ \frac{\pi}{2} - (\alpha + \beta) + \frac{\pi}{2} - \gamma \right]
- l \dot{\theta} \sin \left[ \alpha + \frac{\pi}{2} - (\alpha + \beta) - \frac{\pi}{2} - \gamma \right]
= r \phi \sin - r \phi \cos \phi_{sw}
\]

\[
[\cos (\alpha + \beta + \gamma) \sin (\alpha + \beta + \gamma) - l \sin (\beta + \gamma)]
\left[ \dot{x}_g \; \dot{y}_g \; \dot{\theta} \right]^T + r \phi \sin + r \phi_{sw} \phi_{sw} = 0
\]

Thus, the above rolling condition can be transformed to

---

**Fig. 1 Parameters of a Swedish wheel**

**A. Kinematics of a Three-Wheeled Omnidirectional Robot**

Consider a Swedish wheel mounted on a mobile robot 
with local coordinate frame \( \{R\} \) (\( G - X_s Y_s Z_s \)), as shown 
in Fig. 1, where point \( A \) is the wheel center and other 
geometric parameters are defined as follows. \( \alpha \) is the 
angle of vector \( \overrightarrow{GA} \) relative to \( X_s \) axis, and \( \beta \) is the 
angle between \( \overrightarrow{GA} \) and main wheel axis. The distance 
from \( G \) to wheel center \( A \) is \( l \), and the main wheel’s radius is 
\( r \). And \( \phi \) and \( \phi_{sw} \) are respectively the rotation speeds of 
the main wheel and the passive roller contacting with the 
flat floor.

Assume that the contact point between the Swedish 
wheel and the floor is an instantaneous rotation center, that 
is, it is in pure rolling contact without slipping, then the 
corresponding velocity of wheel center \( A \) is \( r \phi \) along the 
tangential direction as shown in Fig. 1. Thus, the wheel 
center \( A \)’s velocity component along the contact roller’s axis is 
\( r \phi \cos \gamma \).

Assume that the robot’s instantaneous translation 
velocity in terms of local frame \( \{R\} \) is \( [\dot{x}_g \; \dot{y}_g] \), and the 
rotation velocity about \( Z_s \) axis is \( \dot{\theta} \). Then the wheel 
center \( A \)’s velocity vector can also be computed by summing 
the translational velocity components \( \dot{x}_g, \dot{y}_g \), and the 
relative velocity \( l \dot{\theta} \) due to the rotation shown in Fig. 1. 
The wheel center \( A \)’s velocity component along the contact 
roller’s axis can be expressed as \[1\]:
be:

\[
\begin{pmatrix}
\cos(\alpha + \beta + \gamma) & \sin(\alpha + \beta + \gamma) & l \sin(\beta + \gamma)
\end{pmatrix}.
\]

(4)

In this paper we consider the omni-directional robot with three Swedish wheels shown in Fig. 2. The angles \(\alpha, \beta, \) and \(\gamma\) of the three Swedish wheels, \(i = 1, 2, 3\), are shown in Table 1. Based on Eq. (3), we have the three constraint equations for the centers of the three Swedish wheels as follows:

\[
\begin{pmatrix}
\sin(\alpha + \beta + \gamma - \alpha) - \cos(\alpha + \beta + \gamma) - l \cos(\beta + \gamma)
\end{pmatrix},
\]

(5)

\[
\begin{pmatrix}
\sin(\alpha + \beta + \gamma - \beta) - \cos(\alpha + \beta + \gamma) - l \cos(\beta + \gamma)
\end{pmatrix},
\]

\[
\begin{pmatrix}
\sin(\alpha + \beta + \gamma - \gamma) - \cos(\alpha + \beta + \gamma) - l \cos(\beta + \gamma)
\end{pmatrix}.
\]

Assume three same Swedish-90° wheels be used, and the mounting distances be also equal, thus \(\gamma_i = 0\), and \(r_i = r, l, i = 1, 2, 3\). We can obtain the following inverse velocity kinematics equation from Eq. (5):

\[
\begin{pmatrix}
\cos(\pi - \theta) - \sin(\pi - \theta) - l
\end{pmatrix} = \begin{pmatrix}
x_i
\end{pmatrix}.
\]

(6)

From Eq. (5), we can also obtain the forward velocity kinematics equation of the three-wheeled Swedish mobile robot as follows:

\[
\begin{pmatrix}
x_i
\end{pmatrix} = \begin{pmatrix}
\cos(\pi - \theta) - \sin(\pi - \theta) - l
\end{pmatrix}.
\]

(7)

B. Dynamics of a Three Swedish-Wheeled Robot

Consider the mobile robot shown in Fig. 3, where \(G\) is the geometric center with position vector \(r_G = [x_G, y_G]^T\) in terms of inertia frame \(\{I\}\), and \(G'\) is the mass center of the moving platform with relative position vector \(r_{G'G} = [-d_i - d_j]^T\) in terms of robot frame \(\{R\}\). The velocity of point \(G\), \(v_G\), in terms of robot frame \(\{R\}\) can be expressed as:

\[
\begin{pmatrix}
\dot{x}_G \\
\dot{y}_G
\end{pmatrix} = \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix} \begin{pmatrix}
x_i
\end{pmatrix}.
\]

(8)

where \(x_i\) and \(y_i\) are the velocity components of \(G\) along the \(x_i\) and \(y_i\) axes, respectively, and \(\theta\) is the orientation of the platform relative to reference frame \(\{I\}\). Hence the velocity of the mass center \(G'\), \(v_{G'}\), in terms of robot frame \(\{R\}\) can be obtained as:

\[
\begin{pmatrix}
\dot{x}_G \\
\dot{y}_G
\end{pmatrix} = \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix} \begin{pmatrix}
x_i \\
y_i
\end{pmatrix}.
\]

(9)

The total kinetic energy \(T\) of the mobile robot including the translational and rotational parts of the platform and the three Swedish wheels can be computed as below:

\[
T = \frac{1}{2} m_G v_G^T + I_G \dot{\theta}^2 + \sum_{i=1}^{3} m_i (r_{oi})^2 + \sum_{i=1}^{3} I_{oi} \dot{\phi}_{oi}^2
\]

(10)
where \( m_p \) is the mass of the platform, and \( m_w \) is the mass of each wheel; \( I_p \) is the moment of inertia of the platform about \( Z_p \) axis (parallel to \( Z_p \)) through point \( G \), and \( I_i \) is the moment of inertia of \( i \)th wheel about its main axis; \( \dot{\theta} \) is the rotational speed of the platform, and \( \phi_i \) is the rotational speed of the \( i \)th wheel about its main axis; and \( r \) is the radius of each Swedish wheel. Since the mobile robot is assumed moving in a plane, the total potential energy \( V \) is 0. After substituting Eq. (3) into Eq. (10) and some computations, the Lagrangian \( L = T - V = T \) can be obtained as follows:

\[
L = \frac{1}{2} \sum_{i=1}^{3} \left[ \left( \dot{x}_i, \cos(\theta), \sin(\theta) \right)^2 + \left( \dot{y}_i, \cos(\theta), \sin(\theta) \right)^2 + \frac{1}{r^2} \left( \dot{x}_i \cos(\theta) - \dot{y}_i \sin(\theta) \right)^2 \right] + \left[ \frac{1}{r^2} \left( \dot{x}_i \cos(\theta) - \dot{y}_i \sin(\theta) \right)^2 \right] + \left[ \frac{1}{r^2} \left( \dot{x}_i \cos(\theta) - \dot{y}_i \sin(\theta) \right)^2 \right] + \left[ \frac{1}{r^2} \left( \dot{x}_i \cos(\theta) - \dot{y}_i \sin(\theta) \right)^2 \right] + \left[ \frac{1}{r^2} \left( \dot{x}_i \cos(\theta) - \dot{y}_i \sin(\theta) \right)^2 \right]
\]

The dynamics model can then be derived using the Lagrange's equations:

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = \tau_i, \quad i = 1, 2, 3 \tag{12}
\]

where \( q_i \) is the \( i \)th generalized coordinate, and \( \tau_i \) is the \( i \)th generalized force/torque. The generalized coordinate vector is defined as: \( q = [q_1, q_2, q_3]^\top = [x_i, y_i, \theta]^\top \). Refer to Fig. 3, where \( f_i \) is the contact friction force of the \( i \)th Swedish wheel with the floor, the generalized force/torque \( F_i, \quad i = 1, 2, 3 \), can be derived as follows [20]:

\[
F_i = \sum_{j=1}^{3} (r_j - r \sgn(\phi_j) f_j) \frac{\partial \phi_j}{\partial x_i} + \sum_{j=1}^{3} (r_j - r \sgn(\phi_j) f_j) \frac{\partial \phi_j}{\partial y_i} \tag{13}
\]

By Eq. (6), we can obtain:

\[
\frac{\partial \phi_j}{\partial x_i} = \frac{1}{r} \cos(\frac{\pi}{6} - \theta),
\]

\[
\frac{\partial \phi_j}{\partial y_i} = -\frac{1}{r} \sin \theta,
\]

\[
\frac{\partial \phi_j}{\partial x_i} = -\frac{1}{r} \cos(\frac{\pi}{6} + \theta)
\]

Thus,

\[
F_i = \left[ r_j - r \sgn(\phi_j) f_j \right] \left[ \frac{1}{r} \cos(\frac{\pi}{6} - \theta) \right] + \left[ r_j - r \sgn(\phi_j) f_j \right] \left[ \frac{1}{r} \sin \theta \right]
\]

Similarly,

\[
F_i = \sum_{j=1}^{3} (r_j - r \sgn(\phi_j) f_j) \frac{\partial \phi_j}{\partial \theta}
\]

\[
= \left[ r_j - r \sgn(\phi_j) f_j \right] \left[ \frac{1}{r} \cos \theta \right] + \left[ r_j - r \sgn(\phi_j) f_j \right] \left[ \frac{1}{r} \sin(\frac{\pi}{6} + \theta) \right]
\]

After some straightforward computations with equal wheel inertias \( I_1 = I, \quad i = 1, 2, 3 \), the equations of motion of the mobile robot can be expressed in matrix/vector form as:

\[
M(q) \ddot{q} + \mathbf{C}(q, \dot{q}) \dot{q} + \mathbf{J}^T \mathbf{f} = \frac{1}{r} J^T \tau \tag{17}
\]

where \( M = \begin{bmatrix} m_{q_1}, m_{q_2}, m_{q_3} \end{bmatrix} \) is the inertia matrix,

\( \mathbf{C} = \begin{bmatrix} c_{q_1}, c_{q_2}, c_{q_3} \end{bmatrix} \) is the Coriolis and centripetal matrix,

\( \tau = [\tau_1, \tau_2, \tau_3]^\top \), \( \mathbf{f} = [f_1, f_2, f_3]^\top \),

\( \mathbf{S} = \text{diag} \begin{bmatrix} \sgn(\phi_1), \sgn(\phi_2), \sgn(\phi_3) \end{bmatrix} \),

\( m_{11} = m_w + \frac{3}{2} (m_w + \frac{1}{r^2} I), m_{12} = m_{21} = 0, \)

\( m_{13} = m_{31} = m_w (d_1 \sin \theta + d_2 \cos \theta), \)

\( m_{22} = m_w + \frac{3}{2} (m_w + \frac{1}{r^2} I), \)

\( m_{23} = m_{32} = m_w (-d_2 \cos \theta + d_1 \sin \theta), \)

\( m_{33} = m_w (d_1^2 + d_2^2) + \left( I_w + 3 \frac{1}{r^2} I \right), \)

\( c_{13} = m_w (d_1 \cos \theta - d_2 \sin \theta) \)

\( c_{23} = m_w (d_2 \sin \theta + d_1 \cos \theta) \),

and other \( c_{ij} \)’s are all zero, and

\[
\mathbf{J} = \begin{bmatrix} \cos(\frac{\pi}{6} - \theta) & -\sin(\frac{\pi}{6} - \theta) & -1 \\
-\sin \theta & \cos \theta & -1 \\
-\cos(\frac{\pi}{6} + \theta) & -\sin(\frac{\pi}{6} + \theta) & -1 \end{bmatrix}
\]
III. ADAPTIVE CONTROL OF AN OMNI-DIRECTIONAL WHEELED ROBOT

A. Modeling Uncertainty

In reality, the mass of the platform may be carrying a payload, and the contact friction forces are not completely known, thus we can model their uncertainty by letting \( m_b = \hat{m}_b + \Delta m_b \), and \( \mathbf{f} = \mathbf{f} + \Delta \mathbf{f} \), where \( \hat{m}_b \) and

\[
\hat{f} = \frac{1}{3}(\hat{m}_b + 3m)g\begin{bmatrix}\hat{u}_{r,1} & \hat{u}_{r,2} & \hat{u}_{r,3}\end{bmatrix}^T
\]

are the nominal platform mass and gravity constant, respectively. Here \( g \) is the gravity constant, and \( \hat{u}_{r,i}, \ i = 1,2,3 \), are the nominal rolling resistance coefficients of the three wheels. By substituting into Eq. (17), the dynamics of the mobile robot considering uncertainty can be summarized as below:

\[
\mathbf{\dot{M}}\mathbf{\ddot{q}} + J^T\mathbf{\dot{S}}\mathbf{\dot{f}} + (\hat{m}_b + \Delta m_b)\mathbf{H}\mathbf{\dot{q}} + \psi_2 + \psi_3 + J^T\mathbf{S}\Delta \mathbf{f} = \frac{1}{r}J^T\mathbf{\tau}
\]

where \( \mathbf{\dot{M}} \) is the nominal inertia matrix,

\[
\mathbf{\dot{M}} = \text{diag}\begin{bmatrix} \hat{m}_b + \frac{3}{2}(m_b + \frac{1}{r}I) & \hat{m}_b + \frac{3}{2}(m_b + \frac{1}{r}I)
\end{bmatrix},
\]

\[
\mathbf{I}_b + 3I^2\begin{bmatrix} m_b + \frac{1}{r}I \end{bmatrix}
\]

\( \hat{\mathbf{f}} \) is the nominal friction vector, and

\[
\psi_1 = \begin{bmatrix} d_1 & d_2 \end{bmatrix}^T,
\]

\[
\psi_2 = \begin{bmatrix} 0 & 0 \end{bmatrix}^T \hat{\mathbf{f}}(\hat{m}_b + \Delta m_b)(d_1^2 + d_2^2),
\]

\[
\psi_3 = \begin{bmatrix} \hat{\mathbf{u}}_{r,1} & \hat{\mathbf{u}}_{r,2} & \hat{\mathbf{u}}_{r,3} & \Delta m_b \hat{\mathbf{u}}_{r,1} & \Delta m_b \hat{\mathbf{u}}_{r,2} & \Delta m_b \hat{\mathbf{u}}_{r,3} & 0 \end{bmatrix}^T,
\]

\[
\mathbf{H} = \begin{bmatrix} \hat{\mathbf{u}}_{r,1} & \hat{\mathbf{u}}_{r,2} & \hat{\mathbf{u}}_{r,3} & \Delta m_b \hat{\mathbf{u}}_{r,1} & \Delta m_b \hat{\mathbf{u}}_{r,2} & \Delta m_b \hat{\mathbf{u}}_{r,3} & 0 \end{bmatrix}
\]

\[
\begin{bmatrix} \hat{\mathbf{u}}_{r,1} & \hat{\mathbf{u}}_{r,2} & \hat{\mathbf{u}}_{r,3} & \Delta m_b \hat{\mathbf{u}}_{r,1} & \Delta m_b \hat{\mathbf{u}}_{r,2} & \Delta m_b \hat{\mathbf{u}}_{r,3} & 0 \end{bmatrix}
\]

Defining the above uncertainty terms as:

\[
\mathbf{D}_{1,3} = \hat{m}_b \mathbf{H}\mathbf{\dot{q}}, \quad \mathbf{D}_{1,2} = \Delta m_b \mathbf{H}\mathbf{\dot{q}}, \quad \psi_2 + \psi_3,
\]

\[
\mathbf{D}_1 = \mathbf{D}_{1,1} + \mathbf{D}_{1,2}, \quad \mathbf{D}_2 = J^T\mathbf{S}\Delta \mathbf{f},
\]

the dynamics model can be rewritten as

\[
\mathbf{\dot{M}}\mathbf{\ddot{q}} + J^T\mathbf{\dot{S}}\mathbf{\dot{f}} + \mathbf{D}_1 + \mathbf{D}_2 = \frac{1}{r}J^T\mathbf{\tau}
\]

B. Stable Adaptive Control Design

1) Nominal Control Derivation:

In order to design the adaptive control law, the uncertainty terms can be first neglected, i.e., let \( \mathbf{D}_1 = \mathbf{D}_2 = 0 \), and consider the following nominal model:

\[
\mathbf{\dot{M}}\mathbf{\ddot{q}} = \frac{1}{r}J^T\mathbf{\dot{S}}\mathbf{\dot{f}}
\]

Selecting the state vector as

\[
\mathbf{x} = \begin{bmatrix} x_1^T & x_2^T \end{bmatrix}^T = \begin{bmatrix} q^T & \dot{q}^T \end{bmatrix}^T = \begin{bmatrix} x_1 & y_1 & \dot{x}_1 & \dot{y}_1 & \dot{\theta} \end{bmatrix}^T,
\]

the state equation of the nominal system can be written as

\[
\begin{bmatrix} \dot{x}_1 = x_2 \\
\dot{x}_2 = \mathbf{M}^{-1}\left(\frac{1}{r}J^T\mathbf{\tau} - J^T\mathbf{S}\mathbf{\dot{f}}\right) \end{bmatrix}
\]

Using the backstepping method, a stable nominal control law can be obtained as follows \([21]\).

First consider the \( x_1 \) subsystem, \( \dot{x}_1 = x_2 \). Let

\[
\dot{x}_1 = v_1
\]

Where \( v_1 \) is a virtual input variable. Define the tracking error as

\[
e_1 = x_1 - q_d
\]

By considering the Lyapunov function candidate

\[
V_1 = \frac{1}{2}e_1^T \mathbf{K}_1 e_1
\]

where \( \mathbf{K}_1 \in \mathbb{R}^{10\times10} \) is symmetric and positive definite, we have

\[
\dot{V}_1 = e_1^T \mathbf{K}_1 e_1 = e_1^T \mathbf{K}_1 (v_1 - \dot{q}_d)
\]

Thus, we can choose

\[
v_1 = \dot{q}_d - e_1
\]

and obtain

\[
\dot{V}_1 = -e_1^T \mathbf{K}_1 e_1 \leq 0
\]

Hence we know that \( \lim_{t \to \infty} e_1(t) = 0 \), that is, the subsystem is asymptotically stable.

Furthermore, the whole nonlinear system (22) is considered. After introducing new error vector

\[
e_2 = x_2 - v_1
\]

we have

\[
\dot{e}_1 = \dot{x}_1 - \dot{q}_d = x_2 - \dot{q}_d - v_1 + v_1
\]

\[
= e_2 + v_1 - \dot{q}_d = e_2 - e_1
\]

\[
\dot{e}_2 = \dot{x}_2 - \dot{v}_1 = \dot{x}_2 - \dot{q}_d + e_1
\]

\[
= \frac{1}{r}J^T\mathbf{\dot{S}}\mathbf{\dot{f}} - \dot{q}_d + (e_2 - e_1)
\]

Then by considering new Lyapunov function candidate as

\[
\dot{V}_2 = V_2 + \frac{1}{2}e_2^T \mathbf{K}_2 e_2
\]

where \( \mathbf{K}_2 \in \mathbb{R}^{10\times10} \) is symmetric and positive definite, and differentiating \( V_2 \), we have
\[
\dot{V}_2 = e_1^T K_e e_1 + e_2^T K_e e_2 \\
= e_1^T K_e (e_1 - e_1) + e_2^T K_e (e_2 - e_1) \\
+ e_2^T K_e \left[ \frac{1}{r} J^T \tau - J^T S_f \right] - \dot{q}_d + (e_2 - e_1) \\
(33)
\]

Thus, we can choose the nominal control law as
\[
\tau = u \\
= r B (B^T B)^{-1} \dot{M} \left[ \dot{q}_d + M^{-1} B^T S_f \right] - 2 e_1 + e_1 - K_e e_1 \\
(34)
\]

and obtain
\[
\dot{V}_2 = -e_1^T K_e e_1 - e_2^T K_e e_2 = -2V_2 \leq 0 \\
(35)
\]

Since \( \dot{V}_2 \) is negative definite, we know that the equilibrium point \( e = [e_1^T, e_2^T]^T = 0 \) is exponentially stable.

2) Adaptive Control of Robot with Uncertainty:

Revisit the three-wheeled robot dynamics model with uncertainty \( D_1, D_2 \):
\[
\dot{M} \dot{q} + J^T S_f + D_1 + D_2 = \frac{1}{r} J^T \tau \\
(20)
\]

Using the same error vectors as in the nominal control design,
\[
e_1 = q - q_d \\
e_2 = \dot{q} - \dot{q}_d + e_1 \\
(36)
\]

the system error dynamics can be obtained by differentiating Eq. (36) as below:
\[
\dot{e} = \alpha(t, q, \dot{q}) + \beta(q) \left( \frac{1}{r} J^T \tau - D_1 - D_2 \right) \\
(37)
\]

where
\[
\alpha(t, q, \dot{q}) = \begin{bmatrix} e_2 - e_1 \\ - \dot{M}^{-1} J^T S_f - \dot{q}_d + (e_2 - e_1) \end{bmatrix} \\
\beta(q) = \begin{bmatrix} 0 \\ \dot{M}^{-1} \end{bmatrix}.
\]

Since there exists \( \rho_1 > 0 \) and
\[
\psi(q, \dot{q}) = [1.3(\ddot{x}_1 + \ddot{y}_1) + 65.3(\ddot{x}_2 + \ddot{y}_2) + 18.2(\ddot{x}_1 + \ddot{y}_1)(\ddot{x}_1 + \ddot{y}_1) + 0.82(\ddot{x}_2 + \ddot{y}_2)^2 + 0.2] \]
\[
(38)
\]

such that the norms of the uncertainties \( D_{1,2} \) and \( D_1 \) satisfy
\[
\|D_{1,2}\| \leq \psi(q, \dot{q}) \\
\|D_1\| \leq \rho_1, \\
(39)\]

we can design a linear-in-the-parameter function approximator \( F_{\lambda}(x, \dot{x}) \) to compensate for the effect of nonlinear \( D_{1,2} \). Consider the following Lyapunov function in the adaptive control design,
\[
V = V_2 + \frac{1}{W} \dot{w}_c^T \Gamma_d \dot{w}_c + \frac{1}{W} \dot{w}_c^T \Gamma_d^{-1} \dot{w}_c \\
(41)
\]

where \( \Gamma \) and \( \Gamma_d \) are symmetric and positive definite weight matrices; \( \dot{w}_c = \dot{w} - \dot{w}_c \), here \( \dot{w} \) is the optimal vector for \( \dot{w}_c \), and \( \dot{w}_c = \psi_\eta - \psi_d \). The adaptive control law composed of the nominal control (34) and a compensating term \( u_d \) can be selected as follows:
\[
\tau = u + u_d \\
(42)
\]

with
\[
u_d = r J^T \rho_2 \left( \frac{\partial V}{\partial \psi} \beta(q) \right)^T + c_1 \\
+ \frac{r J^T \rho_2 \psi}{\partial \psi} \left( \dot{M} \dot{H} \psi_\eta + F_{\lambda}(x, \dot{x}) - \hat{\eta} \left( \frac{\partial V}{\partial \psi} \beta(q) \right)^T \right) \\
(43)
\]

where \( c_1 > 0, \eta > 0 \).

3) Stability Proof of the Closed-Loop System:

In order to synthesize the compensator for uncertainty \( D_{1,2} \), we can first define
\[
\xi = \frac{\partial V}{\partial \psi} \beta(q)^T \\
(44)
\]

\[
v_\lambda = -k_\psi \psi^2, \quad k > 0 \\
(45)
\]

where \( v_\lambda \) is to be used to approximate \( D_{1,2} \). Since
\[
\xi^T (v_\lambda - D_{1,2}) = \xi^T (-D_{1,2} - k_\psi \psi^2) \\
\leq \psi \xi^T \xi - k_\psi \xi^T \psi^2, \\
(46)
\]

by letting \( a = \sqrt{k} \xi \psi, \quad b = \frac{1}{2} \frac{1}{\sqrt{k}} \) and
\[
-a^T a + 2a^T b \leq b^T b, \\
\]

we have
\[
\xi^T (v_\lambda - D_{1,2}) \leq \frac{1}{4k} \xi^T \xi \\
(47)
\]

By a suitable construction of the function approximator \( F_{\lambda}(x, \dot{x}) \), and letting \( F_{\lambda}(x, \dot{x}, w) \) be the corresponding optimal function approximator, the optimal approximating error can be defined as
\[
w_\lambda = F_{\lambda}(x, \dot{x}) - v_\lambda \\
(48)
\]

Assume that \( w_\lambda \) be bounded, i.e., there exists a \( W > 0 \) such that
\[
|w_\lambda| \leq W. \\
(49)
\]

Since the function approximator can be selected as being linear-in-the-parameter, the approximating error can be expressed as
\[
F_{\lambda}(x, \dot{x}) - D_{1,2} = F_{\lambda}(x, \dot{x}) - F_{\lambda}(x, \dot{x}) + F_{\lambda}(x, \dot{x}) - D_{1,2} \\
= \frac{\partial F_{\lambda}(x, \dot{x})}{\partial \psi} \dot{w}_c + w_\lambda + v_\lambda - D_{1,2} \\
(50)
\]

Taking the time derivative of \( V \), we have
\[ V_\dot{} = \frac{\partial V}{\partial \eta} \left[\alpha(t,q,q) + \beta(q) \left( \frac{1}{r} J^T u \right) \right] + \frac{\partial \gamma}{\partial \eta} \beta(q) \left( 1 - J^T u_D - D_k - D_k^T \right) + \tilde{w} \Gamma^{-1} \dot{w} + \tilde{w}_d \Gamma_d^{-1} \dot{w}_d \]

\[ = -2V_\gamma - \frac{\partial \gamma}{\partial \eta} \beta(q) \left[ \frac{\partial}{\partial \eta} \left( -\frac{\partial \gamma}{\partial \eta} \beta(q) \right) \right] + \dot{w} \Gamma^{-1} \dot{w} + \dot{w}_d \Gamma_d^{-1} \dot{w}_d \]

where \( \sigma, \sigma_d > 0 \), and \( \dot{w} \) and \( \dot{w}_d \) are the best guesses for the unknown parameter vectors \( w \) and \( w_d \), respectively. By letting \( a = \sqrt{\eta} \), we have

\[ \dot{V}_\gamma = -2V_\gamma - \frac{\partial \gamma}{\partial \eta} \beta(q) \left[ \frac{\partial}{\partial \eta} \left( -\frac{\partial \gamma}{\partial \eta} \beta(q) \right) \right] + \dot{w} \Gamma^{-1} \dot{w} + \dot{w}_d \Gamma_d^{-1} \dot{w}_d \]

Using inequality: \(-2a^2 + 2a b \leq -a^2 + b^2 \), we have

\[ -\dot{w} \Gamma^{-1} \dot{w} \leq -\dot{w} \Gamma^{-1} \dot{w} - \dot{w}_d \Gamma_d^{-1} \dot{w}_d \]

Substituting Eqs. (55) and (56) into Eq. (54), we have

\[ \dot{V}_\gamma \leq -2V_\gamma - \frac{\partial \gamma}{\partial \eta} \beta(q) \left[ \frac{\partial}{\partial \eta} \left( -\frac{\partial \gamma}{\partial \eta} \beta(q) \right) \right] + \dot{w} \Gamma^{-1} \dot{w} - \dot{w}_d \Gamma_d^{-1} \dot{w}_d \]

Similarly,

\[ \dot{V}_\gamma \leq -\frac{\dot{w}^2}{2} + \frac{\dot{w}_d^2}{2} \]

Hence, we can choose the parameter adaptation laws as

\[ \dot{w} = -\Gamma \left[ \frac{\partial \gamma}{\partial \eta} \beta(q) \Gamma^{-1} \right] + \sigma \left( \dot{w} - \dot{w}_d \right) \]

\[ \dot{w}_d = -\Gamma \left[ \frac{\partial \gamma}{\partial \eta} \beta(q) \right] \]

where \( \sigma, \sigma_d > 0 \), and \( \dot{w} \) and \( \dot{w}_d \) are the best guesses for the unknown parameter vectors \( w \) and \( w_d \), respectively. By letting \( a = \sqrt{\eta} \), we have

\[ \dot{V}_\gamma \leq -2V_\gamma - \frac{\partial \gamma}{\partial \eta} \beta(q) \left[ \frac{\partial}{\partial \eta} \left( -\frac{\partial \gamma}{\partial \eta} \beta(q) \right) \right] + \dot{w} \Gamma^{-1} \dot{w} - \dot{w}_d \Gamma_d^{-1} \dot{w}_d \]

Using inequality: \(-2a^2 + 2a b \leq -a^2 + b^2 \), we have

\[ -\dot{w} \Gamma^{-1} \dot{w} \leq -\dot{w} \Gamma^{-1} \dot{w} - \dot{w}_d \Gamma_d^{-1} \dot{w}_d \]

Substituting Eqs. (55) and (56) into Eq. (54), we have

\[ \dot{V}_\gamma \leq -2V_\gamma - \frac{\partial \gamma}{\partial \eta} \beta(q) \left[ \frac{\partial}{\partial \eta} \left( -\frac{\partial \gamma}{\partial \eta} \beta(q) \right) \right] + \dot{w} \Gamma^{-1} \dot{w} - \dot{w}_d \Gamma_d^{-1} \dot{w}_d \]

Similarly,

\[ \dot{V}_\gamma \leq -\frac{\dot{w}^2}{2} + \frac{\dot{w}_d^2}{2} \]

Hence, we can choose the parameter adaptation laws as

\[ \dot{w} = -\Gamma \left[ \frac{\partial \gamma}{\partial \eta} \beta(q) \right] + \sigma \left( \dot{w} - \dot{w}_d \right) \]

\[ \dot{w}_d = -\Gamma \left[ \frac{\partial \gamma}{\partial \eta} \beta(q) \right] \]
\[ |e| \geq y_i \left( \frac{d}{2} \right) b_i \]. Similarly, while \(- \frac{\sigma}{2} |\hat{w}|^2 + d \leq 0\), and
\[-\frac{\sigma}{2} |\hat{w}_i|^2 + d \leq 0\), we have \(|\hat{w}| \geq \sqrt{\frac{2d}{\sigma}} \), and
\[|\hat{w}_i| \geq \sqrt{\frac{2d}{\sigma}} \), respectively. Hence,

if \(|e| \geq b_i\), or \(|\hat{w}| \geq b_{i1}\), or \(|\hat{w}_i| \geq b_{i2}\), then
\[V_x \leq 0\] (61)

By properly choosing the controller parameters: \(\rho_1\), \(c_1\), \(\sigma\), \(\Gamma\), \(\Delta\), \(K\), \(K_x\), the constants \(b_i\) and \(b_{i2}\) can be made sufficiently small. Thus the adaptive control system is stable and has good tracking performance.

C. Fuzzy Function Approximator Design

This subsection will construct a fuzzy function approximator using T-S fuzzy systems to compensate for \(v_i = \psi^2\). Since the major variables that affect \(\psi\) are \(x_i\) and \(y_i\), \((x_i + y_i)\) is chosen as the input variable of the fuzzy approximator, and the output variable is the estimate for \(v_i\). In the universe of discourse of the input variable, five fuzzy sets are defined as in Fig. 4. The rule base of the fuzzy approximator is considered as follows:

Rule i: If \((x_i + y_i)\) is \(M_i\), then
\[\hat{F}_i(t) = a_i(x_i + y_i) + b_i\], \(i = 1, 2, \ldots, 5\) (62)

where \(M_i\) is the \(i\)th fuzzy set in the universe of discourse of the input variable \((x_i + y_i)\).

Fig. 4 Membership functions for the input variable

Using singleton fuzzifier, product inference engine, and center average defuzzifier [22], the mapping of the fuzzy approximator is
\[\hat{F} = \sum_{i=1}^{5} \mu_i \hat{F}_i = \sum_{i=1}^{5} \mu_i \left[ a_i(x_i + y_i) + b_i \right] \] (63)

where \(\mu_i = M_i(x_i + y_i)\) is the degree of firing of the \(i\)-th rule’s antecedent. Since \(\sum_{i=1}^{5} \mu_i = 1\), Eq. (63) can be written as:
\[\hat{F} = \sum_{i=1}^{5} \mu_i \hat{F}_i = \left[ \mu_i(x_i + y_i) \right] \mu_i \cdots \mu_i(x_i + y_i) \mu_i \right]. \] (64)

Defining \(\chi^T = [(x_i + y_i) \ 1]\) and \(w_i = [a_i \ b_i]^T\), we have
\[\hat{F} = \left[ \mu_i \chi^T \right] \mu_i \chi^T \cdots \mu_i \chi^T \] (65)

where
\[X = \left[ \mu_i \chi^T \right] \mu_i \chi^T \cdots \mu_i \chi^T \] (66)
is the regressor vector and
\[w = \left[ w_i \ w_i \cdots w_i \right]^T \] (67)
is the unknown parameter vector. And the function approximator \(\hat{F}\) to compensate for the effect of \(D_{12}\) can be written as follows:
\[\hat{F}_a = -k\zeta \hat{F} = -k\zeta Xw \] (68)

Hence, the parameter adaptation law (52) can be rewritten as
\[\dot{\hat{w}} = -I \left[ (e_i K_e M^{-1} (-k\zeta X)) + \sigma (\hat{\hat{w}} - w^0) \right] \] (69)

and the adaptive control law is
\[\tau = rJ^T \left[ \dot{\hat{M}} + \hat{M}^{-1} J^T S \hat{f} + e_i - 2\epsilon \epsilon - K_{12} K_e e_i \right] \]
\[= \rho_1 \hat{M}^T K_e e_i + \epsilon \hat{M} \hat{H}_\psi + \epsilon \hat{H}_\psi \] (70)

IV. RESULTS AND DISCUSSION

In this section, two computer simulation examples are used to illustrate the performance of the proposed adaptive control law for the omni-directional wheeled robot. The first simulation considers the pure translation motion along a rectangular desired trajectory in the x-y plane with fixed orientation, as shown in Fig. 5(b). The platform’s geometric center is planned to move forward 1 m along the \(X_1\) axis from the origin of the inertia frame \(\{I\}\), then leftward 1 m along the \(Y_1\) axis, and then backward 1 m along the \(X_1\) axis, and finally move rightward 1 m along the \(Y_1\) axis and return to the origin. The corresponding desired trajectories of the platform’s geometric center, \(x_{d,1}(t)\) and \(y_{d,1}(t)\) are obtained via cubic spline interpolation [23], and the orientation is kept \(\theta_d(t) = 0\). They are shown as the dashed lines in Fig. 5(a).
The parameters of the mobile robot are selected as follows: $m_b = 15\text{ kg}, J = 0.2\text{ kg\cdot m}^2$. 

Fig. 5 Simulation results of the translational case (a) $x, y$, and $\theta$ responses, (b) Geometric center $(x(t), y(t))$ tracking results, (c) Tracking errors.
\[ I = 4.0378 \times 10^{-3} \text{ kg-m}^2, \quad l = 0.27 \text{ m}, \quad m_u = 0.313 \text{ kg}, \]
\[ r = 0.0508 \text{ m}, \quad \dot{u}_u = 0.3, \quad \text{and} \quad g = 9.8 \text{ m/s}^2. \]

And the adaptive controller parameters are chosen below:
\[ \rho = 0.002, \quad c = 0.00001, \quad \eta = 0.001, \quad K_z = \text{diag}[550 \ 550 \ 2.5], \]
\[ K_2 = \text{diag}[8 \ 8 \ 0.8], \quad k = 6, \quad w^0 = 0_{10 \times 1}, \quad \Gamma = 500I_{10 \times 10}, \quad \Gamma_d = 0.01I_{2 \times 2}, \quad \sigma_d = 0, \quad \text{and} \quad \sigma = 0.2I_{10 \times 10}. \]

In the simulation, we consider the platform having eccentricity with \( d_1 = d_2 = 0.02 \text{ m} \), and the mass has variation \( \Delta m_k = 2 \text{ kg} \). The contact frictions of the wheels are assumed with uncertainty \( \Delta f = [0.05 \ 0.05 \ 0.05] \text{ N} \). The simulation results are shown in Fig. 5. Fig. 5(a) depicts the tracking performance of the \( x \)- and \( y \)-axis translation, and the variation of the orientation angle \( \theta \). The corresponding moving trajectory of the geometric center in the \( x-y \) plane is shown in Fig. 5(b). And Fig. 5(c) shows the tracking errors. We know that the largest tracking errors along the \( x \) and \( y \) axes are about 0.01 m, and the largest orientation \( \theta \) error is about 0.025 rad. The required control torques of the three Swedish wheels is shown in Fig. 5(d). From the results, we know that during the forward (0-3 s) and backward (6-9 s) periods, the control torques of wheel 1 and wheel 3 are nearly symmetric and in different directions. This is because that the wheel plane of wheel 2 is orthogonal to the moving direction, and thus wheel 2 needs nearly no driving torque. The adaptation processes of the uncertainty compensation parameter vectors \( \dot{w}(t) \) and \( \dot{\psi}(t) \) are shown in Fig. 5(e).

In order to know the tracking performance of the suggested control law for the pure rotational case, a second simulation with desired trajectories shown as the dashed lines in Fig. 6(a) is further considered. Using the same controller parameters and considering the same uncertainties as the first simulation case, the control results are shown in
Fig. 6. Fig. 6(a) depicts the tracking performances of the \( x \)- and \( y \)-axis translation, and the rotation angle \( \theta \), and Fig. 6(b) shows their tracking errors. We know that the largest undesired displacement along the \( x \) and \( y \) axes can be kept below 0.01 m, and the largest rotation error is about 0.05 rad. The corresponding control torques of the three Swedish wheels are shown in Fig. 6(c). The adaptation processes of the uncertainty compensation parameter vectors \( \hat{w}(t) \) and \( \hat{\psi}(t) \) are shown in Fig. 6(d).
From the simulation results we know that the suggested control system can obtain the desired 3-DOF direct tracking control objective in the Cartesian space, even the platform is encountered a not small eccentricity uncertainty.

V. CONCLUSIONS

In this paper, complete modeling and stable adaptive fuzzy control in the Cartesian space for a three omni-directional wheeled robot had been considered. Based on the derived dynamics model and considering the platform mass variation, eccentricity, and friction uncertainty, a stable adaptive control law was derived using the backstepping method via Lyapunov stability theory. A nonlinear damping term was also included in the control law to compensate for the estimation error, and the parameters update law with $\sigma -$ modification was considered for uncertainty estimation. Computer simulations were conducted to illustrate the suggested control system performance. Usually, control laws derived by the use of Lyapunov stability theory may be too conservative. The suggested control approach in this paper, derived by the backstepping method, can somewhat reduce the conservatism. Real implementation study using the suggested control law with microcontroller deserves future consideration.

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REFERENCES


NOMENCLATURE

\( d_1, d_2 \) eccentricities
\( D_{i1}, D_{i2}, D_i \) uncertainty matrices
\( e_1, e_2 \) position, velocity tracking errors
\( f \) friction vector
\( \hat{f}, \Delta f \) nominal friction and friction variation
\( F_i \) \( i \)th generalized force/torque
\( F_i^f(x, \dot{w}) \) fuzzy function approximator for \( D_{i2} \)
\( g \) gravity constant, 9.81m/s\(^2\)
\( G \) center of mass of the platform
\( G_{X}\{R\} \) robot frame attached to the platform
\( H \) coefficient matrix of the uncertainty term
\( i_1, j_1 \) unit vectors of world frame \( \{I\} \)
\( i_2, j_2 \) unit vectors of robot frame \( \{R\} \)
\( I \) moment inertia of \( i \)th Swedish
\( J \) Jacobian matrix
\( K_1, K_2 \) position, velocity error weight matrices
\( T \) total kinetic energy
\( l \) distance from origin to \( i \)th wheel’s center
\( L \) \( L = T - V \), Lagrangian
\( m_r, \hat{m}_r, \Delta m_r \) robot mass, nominal mass, and variation
\( m_w \) Swedish wheel mass
\( M, \dot{M} \) inertia matrix and its nominal value
\( O_X, Y, (\{I\}) \) world frame
\( q \) generalized coordinate vector
\( r \) radius of Swedish wheels
\( r_{i1} \) geometric center position in terms of \( \{I\} \)
\( r_{G\gamma} \) relative position \( G \) with respect to \( G \)
\( S \) wheel angular velocity direction matrix
\( u, \dot{u} \) nominal and compensation control vectors
\( v_1 \) ideal function for approximating \( D_{i2} \)
\( v_2 \) virtual input vector
\( V \) robot potential energy
\( V_{2, V} \) Lyapunov functions
\( \hat{w} \) parameters estimate vector
\( \dot{w}_1, \dot{w}_2 \) estimate vector for \( \varepsilon_1 \)
\( \dot{w}_o \) best guess for optimal parameter vector \( w \)
\( \dot{x}, \dot{x}_1, \dot{x}_2 \) state, position, and velocity vectors
\( x_{d1}, y_{d1} \) desired trajectories in \( X_1, Y_1 \) directions
\( \alpha_i \) angle of \( i \)th wheel center vector w.r.t. \( X_R \)
\( \beta_i \) angle of \( i \)th wheel center vector w.r.t. its axis
\( \gamma \) angle of passive roller axis and wheel plane
\( \Gamma, \dot{\Gamma} \) weight matrices of adaptation laws \( \dot{w} \) and \( \dot{w}_i \)
\( \eta \) gain of nonlinear damping term
\( \theta \) desired robot orientation trajectory
\( \mu, \mu_2 \) nominal rolling friction coefficient
\( \phi \) angular velocity vector of Swedish wheels
\( \dot{\phi}_1, \dot{\phi}_2 \) velocity vectors in terms of frames \( \{I\}, \{R\} \)
\( \rho_1 \) coefficient of the upper bound function
\( \rho_2 \) upper bound of \( |D_{i2}| \)
\( \sigma, \sigma_2 \) \( \sigma - \) correction coefficient matrices
\( \tau \) Swedish wheel torque vector
\( X \) regressor vector for \( F_i \)
\( \psi_i \) uncertainty vectors, \( i = 1, 2, 3 \)
\( \psi(q, \dot{q}) \) upper bound function, \( |D_{i2}| \leq \rho \psi(q, \dot{q}) \)